

Retrieval of the Green function having coda in a scattering medium by using the cross-correlation function analysis

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Since seismic interferometry was proposed, there have been many measurements on the Green function retrieval by using the CCF of waves. Most of them are measurements of the average velocity from the peak lag time of the CCF, and there were theoretical models for the CCF of waves in a homogenous medium illuminated by noise sources spatially distributed. Recently, there have been attempts to measure precisely the temporal change in the coda portion of the CCF (or ACF) (e. g. Wegler and Sens-Schonfelder, 2007). Here, we propose a theoretical derivation of the Green function having coda in a scattering medium from the CCF of waves radiated from noise sources.

Small-scale velocity anomalies are distributed in a 3-D medium with a constant background velocity. In the following we call the region in which velocity anomalies are distributed a scattering medium. When we represent each velocity anomaly by a delta function, scattering is isotropic and there is no phase change. According to the first order Born approximation for scalar waves, the retarded Green function can be represented as a sum of a direct wave and single scattering waves (coda waves).

The scattering medium is illuminated by noise sources with stationary white spectrum randomly distributed on sphere with a large radius compared with the dimension of the scattering medium. Waves at receivers in the scattering medium can be written by using the convolution of the retarded Green function and noise. We take the average of the CCF over the ensemble of noises distributed.

In the integral, we take the geometrical factors common to all the terms and precisely evaluate the exponent terms using the expansion formula by using the spherical Bessel functions and spherical harmonic functions. We may write the CCF using the imaginary part of the retarded Green function. (Wapenaar and Fokkema (2006) derived the same formula in a general case by using the reciprocal theorem of the correlation type.) Then, the derivative of the ensemble average of the CCF with respect to lag time is written by the convolution of the antisymmetric Green function and the ACF of the noise source.

If many scatterers represented by delta functions are distributed, the retarded Green function has coda waves, of which the envelope decays with lapse time increasing according to the first order Born approximation. The coda envelope can be well approximated by the single isotropic scattering model (Sato,1977).

Here, we have theoretically shown that the Green function can be retrieved from the time derivative of the CCF when the finite scattering medium is illuminated by noise sources on a sphere with a large radius. This model gives a solid base for the practical analysis of the coda portion of CCF or ACF. It will be necessary for us to examine the following cases: there are scattering terms of higher orders, noise sources are distributed uniformly in the scattering medium, and so on.