Fractal analysis of experimentally unstable flow in a conduit

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Heterogeneous ejecta such as banded pumice are commonly found in pyroclastic fall and flow deposits. The banded pumice is interpreted being formed by magma mixing in a conduit. Mechanisms of magma mixing in a conduit were explained by analogue experiments on flow of two liquids where low viscosity liquid occupied center of a pipe, and critical conditions of mixing were estimated (Koyaguchi, 1985; Blake and Campbell, 1986; Freundt and Tait, 1986). Blake and Campbell (1986) revealed entrainment of outer flow into inner flow was promoted as Reynolds number of the inner flow increased with time. Freundt and Tait (1986) showed that conditions of occurrence of the instability depended on the Reynolds number and viscosity ratio between the two liquids. When the viscosity ratio of the two liquids was large, the instability occurred at low Reynolds number. However, in previous analogue experiments, degree of mixing of two liquids has been uncertain. In this study, we conducted analogue experiments on the magma mixing in a conduit, and evaluated the degree of mixing and evolution of the flow with time using fractal analyses.

In the experiments, we used an acrylic apparatus composed by a tank with dimensions of 60 mm$*$60 mm$*$80 mm and a pipe with an inner diameter of 10 mm and length of 600 mm. The pipe was connected to the bottom of the tank. We attached a rubber tube with stopcock to the tip of the pipe to prevent outflow of the two liquids before the experiments. First, we filled the pipe with liquid 2 (dense, highly viscous, clear, and colorless). We closed the stopcock and poured liquid 2 in the tank up to 30 mm thick. When the surface of liquid 2 stabilized, we poured liquid 1 (dyed liquid whose density and viscosity are lower than those of liquid 2) on it up to a height of 30 mm. Each experiment was started by opening the stopcock and allowing the liquids to flow downward under gravity. We recorded the flow using video camera, and measured fractal dimension of the complicated interface of the two liquids as indicator of mixing. In this measurement, we used the box counting method using 2-D pictures.

At the beginning of the experiments, only liquid 2 was drained from the tank into the pipe, but subsequently, when liquid 1 fell to a critical depth in the tank, liquid 1 was sucked into the pipe through the center of liquid 2. Initially, liquid 1 exhibited a thin stream in the pipe. However, the stream evolved into beads shape with time. Size of the individual bead increased with time, and finally liquid 1 occupied most of the pipe. We observed the entrainment of liquid 2 into liquid 1 during growth of the beads. Reynolds number of liquid 1 increased with time. However, the Reynolds number was kept at constant during development of the size of beads. Then the Reynolds number rapidly increased after liquid 1 occupied most of the pipe. Relationship between the critical Reynolds number of liquid 1 ($Re_{critical}$) where instability of liquid 1 occurs and viscosity ratio of the two liquids (v_2/v_1) was described as the following equation: ($Re_{critical}$) = 4.0$*$10^4$*$(v_2/v_1)^a. Where a is nearly -0.6. This relationship is similar to the results of Freundt and Tait (1986). When liquid 1 exhibited a thin stream, fractal dimension (D) of the interface of the two liquids showed minimum value (D = 1.01-1.02), and it reached maximum value (D = 1.04-1.06) in which the fine beads were formed. Then fractal dimension decreased with the growth of the size of a bead. We conclude that when the fractal dimension becomes the maximum value, contact area of the two liquids much increases and ability of the entrainment of liquid 2 into liquid 1 also increases. However, when the size of the beads is large, the fractal dimension decreases and the entrainment of liquid 2 is suppressed. In this stage, the engulfed liquid 2 mixes with liquid 1 in the center flow of liquid 1.