

Numerical tests of the IDO schemes for solving the Vlasov-Poisson equation system including collision and source/sink terms

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Vlasov equation, which is the collision-less and no dissipation limit in the Boltzmann equation and solves the distribution function in phase space, has been widely used in studying various linear and non-linear dynamics in fusion, space and astrophysics plasmas. The Maxwell equation and/or Poisson equation in the electrostatic limit is incorporated in order to self-consistently determine the collision-less plasma dynamics in electromagnetic fields. The numerical methods to solve the Vlasov equation keeping high accuracy and stability with less numerical dissipation are of specifically importance. Furthermore, these methods are expected to be applied to the Fokker-Planck equation with source/sink terms, which are necessary to investigate long time scale behavior dynamics of plasmas.

The splitting scheme by separating the convection dynamics along each coordinate in phase space and the CIP scheme which interpolates the distribution function not only by the grid values, but also by its spatial derivative. However, these methods based on the semi-Lagrange approach are hardly applied to the problems including dissipation and source/sink terms.

Recently, we have applied the Interpolated Differential Operator (IDO) scheme and its conservative form referred to as the IDO-CF one [1], which are developed in CFD area, in solving the Vlasov-Poisson equation system and found that these methods are superior in capturing fine scale structures with less phase errors [2]. They are based on the Euler method, which is easy to apply to other non-advection terms, e.g. collision and source/sink terms. Especially in the IDO-CF2 scheme, it introduces the interpolation functions constructed not only from the grid values of the distribution function, but also from the integrated value between grids, so that the total mass is rigorously conserved. However, note that the IDO-CF2 scheme utilizes the second order interpolation function, which is lower order than that of CIP and the conventional IDO schemes (usually the third order). As a method to increase the accuracy of the IDO-CF2 scheme, a new scheme referred to as the IDO-CF3 has been developed by Aoki et al. [3] and applied to the fluid equation system. This method employs the third order upwind interpolation function, but realizing the less dissipation.

Here, we apply the IDO-CF3 scheme to solve the Vlasov-Poisson equation system. We sample the problem of bump-on-tail (BOT) instability and test the numerical accuracy and efficiency by comparing with other schemes (CIP, IDO and IDO-CF2). As a measure to check the numerical accuracy, we investigate the entropy dynamics during the coalescence process of potential vortices where the entropy is hardly conserved due to the appearance of fine scale structures smaller than mesh size. We found that the higher wave number components of the electrostatic potential can be captured with less dissipation, so that the increase of the entropy is suppressed to be smaller value than those of other schemes. We also successfully utilize the IDO-CF3 schemes including a collision term (i.e. the Fokker-Planck equation) with source/sink terms. Specifically, we

investigated the system where the BOT and 2-stream instabilities coexist which happens in the case with relatively strong source/sink terms, but with weak collision effect.

References

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