

## Development of a new numerical matching technique for resistive MHD stability analysis

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Resistive MHD modes, such as tearing modes, can be unstable in magnetized plasmas if there exists a place where the small plasma inertia and the electrical resistivity can play a role since the wave number vector of Alfvén wave is perpendicular to the magnetic field which results in vanishingly small tension of the magnetic field. This place is called a singular point. One of the major approaches to study this situation is the asymptotic matching method (boundary layer theory)[1]. In the inner layer around the singular point, the governing equation is simplified because the inner layer is thin even if we take inertia and resistivity into account. On the other hand, we solve the so-called Newcomb equation, where the inertia and resistivity are neglected, in the outer region or a region except for the inner layer. The dispersion relation of the resistive MHD modes can be obtained by solving the inner-layer equation, of which boundary conditions are given by solving the Newcomb equation.

However, this asymptotic matching method has several difficulties:

(1) The asymptotic matching method is not applicable if the Newcomb equation has an irregular singular point, since we cannot construct a Frobenius series solution required in the matching procedure. This situation can occur in one of the advanced operations of magnetically confined fusion plasmas, and therefore this difficulty is very important.

(2) Even in a situation where the asymptotic matching method is applicable theoretically, the result of the numerically-performed asymptotic matching is quite sensitive to the accuracy of the numerical solution and the grid-points arrangement near the singularity[2].

(3) The inner layer has a vanishingly thin width when the resistivity is vanishingly small.

However, the width of the inner layer is mapped onto an infinitely wide region using the smallness of the resistivity. Then the solution to the inner-layer equation becomes non-square-integrable, and therefore we require significantly careful treatment[2].

We have developed a numerical matching technique which utilizes an inner layer with finite width in order to resolve those difficulties[3]. The idea to use an inner layer with finite width itself was developed in [4] for Alfvén wave without resistivity. The essential difference of our study with [4] is the inclusion of the resistivity, which makes the order of differentiation of the inner-layer equation four, instead of two in the absence of the resistivity. Thus we need to select appropriate two independent solutions among four so that the solutions in the inner layer can be matched onto the solutions in the outer region. Since the resistivity does not play a role in the outer region, the electric field parallel to the magnetic field must be zero. We found that the numerical matching succeeds if we impose smooth disappearance of the parallel electric field. A boundary condition of the third kind is imposed on the stream function of perturbed velocity field in practice. We will present the details of the formulation and some applications on the stability analysis of cylindrical fusion plasmas[3].

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