

Stability and propagating direction of finite amplitude Boussinesq thermal convection in a rotating spherical shell

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Global thermal convection is considered to occur in stellar interiors, the atmospheres of the giant planets, and the fluid cores of the terrestrial planets, due to the internal heat sources and/or the external cooling. The problem of Boussinesq thermal convection in a rotating spherical shell is one of the most fundamental frameworks to investigate characteristics of these phenomena.

The onset of thermal convection in a rotating spherical shell has been investigated analytically and numerically for half a century. The critical parameters and the critical convection patterns has been revealed for a wide parameter range. Finite-amplitude thermal convection in a rotating spherical shell has also been studied through numerical time integrations in a certain range of parameters, but global behaviour of the solutions is not well understood because it is difficult to survey a wide range of the parameter space systematically, due to the limit of numerical time integrations.

In this study, in order to understand fundamental properties of thermal convection in a rotating spherical shell, we explore finite-amplitude thermal convection solutions by the Newton method rather than numerical time integrations, and examine their stability systematically. We focus our attention on the cases with moderate rotation rate of the shell, where only a relatively low spatial resolution is required, in order to save computer resources and to be able to perform large size systematic array calculations.

The radius ratio of the inner and the outer spheres and the Prandtl number are fixed to be 0.4 and 1, respectively, which are commonly used values in the previous studies. The Taylor number, which is proportional to the rotation rate squared, is varied from 52^2 to 500^2 , and the Rayleigh number is increased from the critical values to about 1.3 times the critical values. In this parameter range, it has already been revealed that when the Taylor number is small a critical convection pattern propagates in the retrograde direction, whereas it propagates in the prograde direction when the Taylor number is large enough. However, the switching mechanism of the propagating direction is not well understood, nor the transition of the convection patterns associated with this switching. Moreover, the existence of a finite-amplitude propagating solution and its phase speed are not clear yet.

First, we investigate the transition of the critical convection patterns and their phase speeds in detail, as the Taylor number increases. It is found that the switching of the phase speed does not occur abruptly but continuously. The structures of convection patterns also change continuously. When the Taylor number is small, the vortex tubes with horizontal structure described by the spherical harmonics Y_l^1 are bent along the spherical shell, while they gradually align in the direction of rotating axis as the Taylor number increases. The switching of the phase speed can be interpreted by expansion and contraction of vortices arising from their meridional structure.

Second, we seek for finite-amplitude propagating convection solutions at supercritical range of the parameters and examine their stability. It is revealed that the finite-amplitude propagating solutions stably exist when the Rayleigh number is between the critical value and about 1.3 times the critical value. Their phase speeds increase continuously as the Rayleigh number increases when the Taylor number is small, however, the phase speeds decrease continuously when the Taylor number is large. In particular, when the Taylor number is between 340^2 and 500^2 , the phase speed change from positive (prograde) to negative (retrograde) values as the Rayleigh number increases. The decrease of the phase speed can be interpreted as an advection by mean zonal flows induced by the nonlinear effects of the thermal convection.

Keywords: bifurcation, phase speed, mean zonal flow