

## Approximate solution estimating the distribution of Young's modulus nearby mode I crack from its shape

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We derived an approximate solution estimating the distribution of Young's modulus nearby mode I crack from its shape.

Shapes of the mode 1 crack (e.g., dike and mineral veins) observed in field differ from theoretical shape estimated under the condition of constant overpressure in an isotropic homogeneous elastic medium. As its cause, we can cite (1) the overpressure acting on the wall of the crack is not constant, (2) the medium is not isotropic homogeneous material or (3) superposition of both effects of them.

In general, the biharmonic function (stress function) does not satisfy the governing equation of non-homogeneous material. Thus, solutions and their deriving methods for the non-homogeneous material are more complex than them for homogeneous material. As shown in Ang and Clements (1987), in this study we assumed that the stress function would be described by a series and attempt to solve the Navier's equation by the Fourier transform. Then, we obtained strict solutions for forward modeling. However, it is difficult to estimate the distribution of Young's modulus nearby a crack from its shape by inverse analysis based on this strict solution for the forward modeling. We attempt to obtain the approximate solution that is directly applicable to inverse modeling, by reducing order of the series describing the stress function. To estimate an outline of inhomogeneity of the medium from shape of dikes and mineral veins plays an important role for setting an initial value for more detailed discussions and forward modelings of their propagation paths and overpressure conditions in the surface.

Because variation in Poisson's ratio normally does not play a significant role in determining magnitudes of displacement in the crack problem, we assumed Poisson's ratio is constant. On the other hand, it was assumed that Young's modulus varies only along the crack. In the field, we find often outcrops of which dikes intrude stratum consisting of some layers which may have an inherent elastic constant. We employed the Fourier cosine series as function describing the distribution of Young's modulus, because the Fourier cosine series can describe an arbitrary function including discontinuity. Then, we derived the solutions for forward and inverse modelings. As simple examples of the inverse analysis, we attempted to estimate the distribution of Young's modulus from shape of the crack given analytically. As a result, the estimated distribution of Young's modulus was found to be correct, although a small amount of noise was present.

[Reference] Ang, W. T., and Clements, D. L., 1987, *Int. J. Solids Structures*, 23, 1089-1104.

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