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Mean-zonal-flow generation in Boussinesq thermal convection in a rotating spherical shell

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Global thermal convection is considered to take place in stellar interiors, the atmospheres of the giant planets, and the fluid cores of the terrestrial planets in the presence of the internal heat sources and/or the external cooling. The Boussinesq thermal convection in a rotating spherical shell, which is one of the most fundamental frameworks to investigate characteristics of these global phenomena, was proposed by Chandrasekhar half a century ago, and has been investigated extensively. However, fundamental properties of its bifurcation structure including the convection patterns bifurcating at the critical point and their stability, are not well understood yet.

Recently we studied the stability and the bifurcation structure of (nonlinear) traveling waves propagating in the longitudinal direction, and found that when the ratio of inner and outer radii is 0.4, the Prandtl number is 1 and the Taylor number is from 52^2 to 500^2, the traveling wave TW4s, which have four-fold symmetry in the longitudinal (azimuthal) direction, bifurcate supercritically at the critical point and are stable for $\mathrm{Ra}_c < \mathrm{Ra} < 1.2\ \mathrm{Ra}_c$–$2\mathrm{Ra}_c$, where Ra and Ra_c are the Rayleigh number and its marginal stability value[1]. The propagating direction of the traveling wave changes from retrograde to prograde on the neutral stability curve as the Taylor number is increased. However, as the Rayleigh number is increased, the direction is found to change again from prograde to retrograde except for the case of small Taylor number.

The change of the propagating direction on the neutral stability curve is explained by the elongation and contraction of vortices[2], while the the direction change with increasing Rayleigh number is interpreted as an advection of the convection cell by the retrograde mean zonal flow generated by the nonlinear effect of the thermal convection.

Here we study the generation mechanism of the mean zonal flow which is crucial for the longitudinal propagation of the finite-amplitude traveling wave, employing the weakly nonlinear analysis[3]. We deal with the parameter region of the supercritical bifurcation of TW4s solution, where the ratio of the inner and outer radii is 0.4, the Prandtl number 1 and the Taylor number from 52^2 to 860^2.

We linearize the governing equation around the state of rest in the rotating frame of reference, and obtain the critical mode. We then calculate the secondary mean flows by substituting the critical mode into the nonlinear terms. There are four nonlinear terms in the governing equations, which we classify into three groups as follows:

(1) the longitudinal velocity component of the Navier-Stokes equation,
(2) the colatitudinal and radial velocity components of the Navier-Stokes equation,
(3) the energy equation.

Each nonlinear term generates the mean zonal flow in the following mechanisms:

(1): the meridional transfer of the angular momentum by the Reynolds stress generates the mean zonal flow,
(2): the Coriolis force against the mean meridional flow induced by the Reynolds stress generates the mean zonal flow,
(3): the Coriolis force against the mean meridonal flow induced by the latitudinal gradient of the secondary mean temperature disturbance, which is caused by the convective heat transfer, generates the mean zonal flow.

We numerically evaluate the mean zonal flow generated by the nonlinear terms (1)–(3), and find that the nonlinear term (3) generates the strong retrograde zonal flow near the middle of equator, which is observed at large Taylor number, several times stronger than those by the nonlinear terms (1) and (2).

[1] K. Kimura, S. Takehiro and M. Yamada, RIMS Kokyuroku 1724, pp.200-211 (2010)
[2] S. Takehiro, J. Fluid Mech., Vol.659, pp.505-515 (2010)
[3] S. Takehiro, Y.-Y. Hayashi, Geophys. Astrophys. Fluid Dynamics, Vol.90, pp.43-77 (1999)

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