

MTT033-P02

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## Designing method of map projections balancing area and angular distortion

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### 1. Introduction

There is no map projection without distortion. Therefore, many map projections have been proposed. Map projections are designed and evaluated by accuracy or appropriateness of area, angle, length, azimuth, etc. However if we focus on infinite area and difference form congruence, area and angle are enough elements to evaluate map projections. Seeking global map projections which balancing area distortion and angular distortion, this presentation propose designing method of such map projections, where. The earth is assumed a share.

Less attention has been directed on distortions of poles; many map projections have expressed a pole as line. This fact means that we have less interest to express poles, where human activities are very low, accurately. However many explorers are launched and many maps of extra-earth planets are produced recently. Because there are no human activities on the planes everywhere, disregard of distortions of poles are not appropriate. Therefore this study intends to balance area and angular distortion on the globe including poles.

### 2. Definition of area distortion and angular distortion

Angular distortion is defined as  $S'/S$ , and angular distortion is defined as  $A'-A$  in cartography, where  $S$  and  $S'$  are area before and after the map projection, and  $A'$  and  $A$  are angle before and after the map projection. Area distortion by this definition shall not be an object to be minimized. Angular distortion by this definition is also not appropriate object to be minimized, because transformation on 1 degree to 2 degrees and 100 degrees to 101 degrees are equally evaluated.

If some quantity  $X$  changed to  $X'$ , distortion is usually defined as  $(X'-X)/X$ . Therefore area and angular distortions are evaluated based on  $E_{S0}=(S'-S)/S$ ,  $E_{A0}=(A'-A)/A$  in this study. If Tissot's indicatrix is available,  $E_{S0}=m_1m_2-1$ , where  $m_1$  and  $m_2$  are major axis and minor axis of Tissot's indicatrix. The maximum of  $|E_{A0}|$  on a point is  $E_A=m_1/m_2-1$ , which is for infinite small angle at the minor axis.

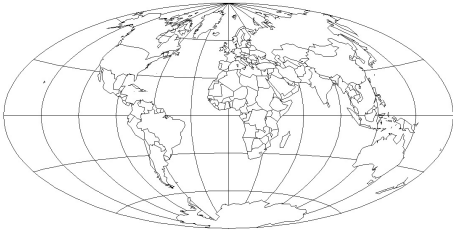
### 3. Evaluation criterion

Values of  $|E_{S0}|$  are different between  $k$  times expansion and  $1/k$  times reduction. Therefore  $E_S=\max(S'/S, S/S')-1=\max(m_1m_2, 1/m_1m_2)-1$  is introduced to evaluate  $k$  times expansion and  $1/k$  times reduction equally. This presentation propose weighted sum of square of  $E_S$  and  $E_A$  for evaluation of distortion, and minimize  $L =$  spherical mean of  $(w_S E_S^2 + w_A E_A^2)$  over the globe, where  $w_S$  and  $w_A$  are weight for area and angular distortion and  $w_S + w_A = 1$ . If  $w_S = w_A$ , then 2 times conformal expansion, 1/2 times conformal reduction, and a equal-area transformation of 2-times-expansion in height and 1/2-times-reduction in width are evaluated equally, which correspond to  $L=4.5$ . Suitability of weighted square sum shall be subject of further study.

### 4. Example

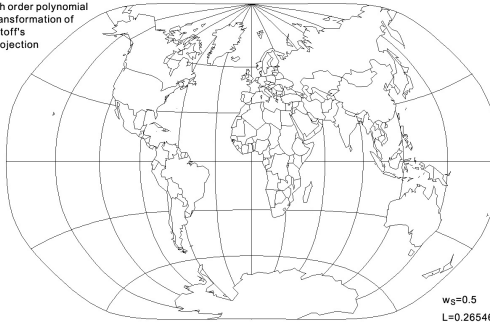
The figure is a result of the preliminary study, which minimizes  $L$  among 9th order polynomial transformation of the Aitoff's projection; step of numerical integration is 5 degree.

Aitoff's projection



$L=0.960981$   
 $(w_0=0.5)$

9th order polynomial transformation of Aitoff's projection



$w_0=0.5$   
 $L=0.265485$

Keywords: map projection, design, area distortion, angular distortion, balance