Implementation of a fast Poisson solver into MHD and PIC simulations

Yosuke Matsumoto

1 Namekawa University Solar Earth Environment Research
1 STEL, Nagoya University

Poisson equation appears in various situations in numerical simulations of space plasmas. For example, the electrostatic potential in the ionosphere is obtained by solving the Poisson equation with the field-aligned current given by the global MHD simulation of the magnetosphere.

The equation is generally solved by iterative solvers. Successive Over-Relaxation (SOR) and Conjugate Gradient (CG) methods are of well used solvers, but the number of iterations increases as \( N^{3/2} \) as the system size (\( N \times N \)) increases. Thus a fast Poisson solver is necessary in a large-scale numerical simulation.

In this presentation, an implementation of Multigrid poisson solver into a MHD simulation code and its application to the global MHD simulation are shown. The solver is implemented in order to remove the monopole of the magnetic field that arises due to the numerical discretization. This enables us to examine a large scale global MHD simulation with keeping \( \text{div}(B) \) negligibly small. The multigrid solver is also applied to the PIC simulation code in which Maxwell equations are solved implicitly. This enables us to simulate under larger \( c/\nu_{\text{th}} = \omega_{pe}/\Omega_{ge}/\sqrt{\beta} \) parameters with a large time step.

Keywords: PIC simulation, MHD simulation, Poisson equation, Multigrid, SOR method