The HLLD approximate Riemann solver in Lagrangian coordinate system

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The magnetohydrodynamic (MHD) equations are the most important equations to describe macroscopic dynamics of plasmas. Since the ideal MHD equations are nonlinear hyperbolic conservation laws, discontinuous solutions such as fast / slow shocks are often generated in nonlinear simulations. Therefore, shock capturing schemes for MHD have been increasingly developed in last decades. In particular, several approximate Riemann solvers \cite{1,2} have been successfully proposed and applied for physical simulation studies since mathematical properties of hyperbolic conservation laws are reflected within those numerical schemes. In those schemes, well-behaved numerical solutions are obtained integrating exact or approximate solutions of the Riemann problem at cell faces over each cell volume in fixed Eulerian grids. However, some problems such as strong shock interactions are difficult to solve on a uniform Eulerian grids although those may be suitable for Lagrangian-type schemes. In this paper, we find an approximate solution of the Riemann problem for MHD in Lagrangian coordinates due to a Lagrangian-type approximate Riemann solver for MHD. Since the MHD equations can be rewritten in another conservative form of the equations in Lagrangian mass coordinates \cite{3}, we can apply an HLL-type approximation in Eulerian grids that is constructed based on conservation laws over the Riemann fan. Particularly, we obtain an HLLD-type approximate Riemann solution \cite{2} where the physical state is assumed to be divided by five-waves in Lagrangian mass coordinates. Mathematical properties of the approximate solution are discussed in detail.

\begin{thebibliography}{9}
\bibitem{1} Brio, Wu, J. Comput. Phys., 75 (1988), 400
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