Riverbed degradation causes many engineering and environmental problems. For example, it destabilizes basements of river structures, and sometimes damages habitats of aquatic organisms because of severe local scours. In the Ishikari River in Asahikawa, the bedrock is exposed due to riverbed degradation. It is suspected that the bed degradation is caused by a decrease in sediment supply from the upstream region by flood control facilities such as dams, or an increase in bed shear stress due to the reduction of the width of low-water channels in river improvement works.

Though it is generally said that bed aggradation destabilizes the flat bed while bed degradation does the opposite, no studies have been done on the behavior of bars under non-equilibrium conditions to the authors’ knowledge. Most of previous studies focus on bars only under equilibrium conditions. The existing linear stability analysis assumes that the river bed is in equilibrium, and that bed slope and flow in the base state are in normal flow conditions.

In this paper, linear stability analysis of bars under weakly non-equilibrium conditions is performed by the use of the WKBJ method. Bed aggradation or degradation is assumed to be sufficiently slow compared with bed evolution due to bed instability, and its non-dimensional speed is used as a small parameter. We introduce two spatial variables with two different length scales: the length scale of aggradation or degradation, and that of bar wavelength. The former is assumed to be much larger than the latter. By solving the base state problem, it is found that one-dimensional base flow is a function of the slowly varying spatial variable. For instance, the flow velocity is accelerated in the streamwise direction, and riverbed has an upward-convex shape under degradation. In the perturbed problem, we expand all the variables with two small parameters: the non-dimensional aggradation/degradation speed, and the amplitude of perturbations. Substituting the asymptotic expansions into the governing equations, and comparing each order of the two small parameters, we obtain base state equations and perturbed equations with the weakly non-equilibrium effects. Solving all the equations with appropriate boundary conditions, we obtain the growth rate of perturbations including the non-equilibrium effects, and instability diagrams for several values of non-dimensional aggradation or degradation speeds.

The analysis shows that, under degradation, the unstable region in the instability diagram is reduced, implying that there is a tendency of the flat bed being stable. In the unstable region, the dominant bar wavelength increases. Meanwhile, under aggradation, the unstable region in the instability diagram is expanded, which means that there is a tendency of the flat bed being unstable. In the unstable region, the dominant wavelength is reduced, implying that bars with short wavelengths are formed. In addition, alternate bars are more affected by bed aggradation/degradation than multiple bars are done.

Keywords: linear stability analysis, alternate bars, aggradation, degradation, sediment supply, WKBJ method