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Constitutive law of rock rheology with fractional order derivative

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Flow law of rocks, a constitutive law for anelastic behavior of rocks, has long been investigated and established based on the experimental and theoretical studies of microstructures in rocks and minerals and played significant roles in developing the framework of the plate tectonics or geodynamic simulations. However, the flow law cannot express the transient behaviors, i.e., the responses to the sudden change in stress or strain rate. Though some empirical laws have also been proposed for the transient behaviors such as Andrade creep law, there are few studies sufficient to explain the mathematical and thermodynamic background for the constitutive equation of transient behavior. Recently, Yajima and Nagahama (2010, J. Phys. A: Math. Theor.) derived a generalized constitutive equation for viscoelastic behavior from an general energy function in terms of differential geometry with fractional order derivative in which the order of derivative is extended from natural number to positive real number and intrinsically includes the effect of time delay. When the strain is differentiated by deformation time in the fractional order, the equation corresponds to the definitional equation of the Boltzmann superposition principle, the fundamental principle of viscoelastic behavior explaining the memory effect. Also the general constitutive law represents the behavior between Hookean elasticity and Newtonian viscosity and involves any type of viscoelastic models such as Maxwell, Kelvin-Voigt and Zener models. Using a reduced equation of the generalized constitutive law, we analyze the experimental data of high-temperature deformation of rocks such as marble, halite and lherzolite and decide the order of the fractional derivative. These rocks exhibit temporal power-law scalings in the relaxation modulus (the ratio of stress to strain, approximately), so the exponent corresponds to the order of the fractional derivative. The orders can be transformed into the exponent of stress in the flow law, and the constitutive law can express both transient and steady-state behaviors. The orders range from 0.04 to 0.13 (the stress exponent, from 7.5 to 25.0) for the transient behaviors and from 0.14 to 0.25 (the stress exponent, from 4.1 to 7.1) for the steady-state behaviors. The exponent of stress in the flow law depends on the deformation mechanisms such as diffusion and dislocation creeps, so we suggest that the order of the fractional derivative is an important parameter connecting the macroscopic time delay to microscopic deformation mechanisms.

Keywords: viscoelasticity, fractional order derivative, rheology, flow law, transient behavior, temporal fractal property