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## Estimation of Underground Structure with Pseudo-Inverse Matrix Calculation

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To estimate the underground structure, we usually use the observational data of gravity anomaly. We can write the relation between the density anomaly underground and the gravity anomaly at the surface as  $Ax=b$ , where vector  $x$  represents the density anomaly, vector  $b$  is the gravity anomaly, and matrix  $A$  is defined by Newton's law of universal gravitation, respectively. This relation is linear, and the reconstruction of density structure from the gravity anomaly can be expected with matrix inversion.

To perform the matrix inversion, we introduce the Moore-Penrose pseudoinverse  $A^+$ , since the number of observation data  $N$  and the number of model point  $M$  are usually different. The theory shows that matrix  $A$  can be decomposed as  $A = USV^T$ , where matrix  $U$  and  $V$  are orthogonal matrices, and  $S$  is a diagonal matrix, whose diagonal elements are singular values  $s_i$ . The Moore-Penrose pseudoinverse  $A^+$  is defined as  $A^+ = VS^+U^T$ , where  $S^+$  is the diagonal matrix, whose diagonal elements are  $1/s_i$ .

In the calculation of matrix  $V$ , the eigenvectors calculation of  $A^T A$  is needed. It is usually performed by QR algorithm, whose computational cost is of the order of  $O(M^3)$ . Since this part is the most time consuming part in the program, we adopt the I-SVD algorithm instead of traditional QR algorithm. I-SVD algorithm is a good algorithm and the computational cost is  $O(M^2)$ . Unfortunately, the next time consuming part of pre-transformation procedure is  $O(M^3)$ . So, the total computational cost remains  $O(M^3)$ .

The matrix inversion, in the terms of underground estimation, is sometimes ill-conditioned. In such case, a round-off error becomes important. Therefore, we introduce the multiple precision arithmetic or arbitrary-precision arithmetic library in our program. The round-off error can be minimised. Since we can easily switch off this option in the program, the comparison between the multiple precision calculation and the usual double precision calculation with the same program is also easy. Various tests show that the introduction of approximate pseudoinverse, whose diagonal elements less than the noise level in the observation data are replaced by 0, gives good results. The optimisation of computational speed by minimising the multiple precision calculation and the evasion of the ill-conditioned problem by introducing an new physics, such as the magnetic data, are the future work.

Keywords: density structure, gravity survey, inversion, pseudoinverse