

The theory of the unperturbed and perturbed rotational motion of celestial bodies in the Sadov and Kinoshita variables

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The analytical theory of the unperturbed and perturbed rotational motion of a rigid celestial body (planet or satellite) is developed. As the unperturbed motion it is accepted and developed the free rotational motion of a rigid triaxial body with unequal principal moments of inertia (the case of the Euler-Poinsot). The body has any triaxiality and arbitrary dynamic oblateness. Also another class of celestial bodies ? non-rigid, and weekly deformed, with a variable geometry of the densities. A new theory of the unperturbed rotational motion of celestial bodies, which takes into account own rotational deformation of the body, is constructed and developed (Chandler-Euler motion, Barkin Yu., 1998). In both cases, the unperturbed rotational motion is a motion on the Euler-Poinsot. However, for weekly deformed celestial body this motion is characterized not by its real moments of inertia, and some the changed constant moments of inertia, taking account of its tidal deformations caused by its own rotation.

Hamiltonian formalism is developed and the unperturbed rotational motion of rigid and weekly deformed celestial bodies are described by <<action-angle>> variables in the form introduced in the well-known works of Yu. Sadov (1972), H. Kinoshita (1977) and Yu. Barkin (1992, 1998). The of paramount importance for the study of the perturbed rotational motion of the planet (satellite) in the gravitational field of the perturbing bodies has the construction of expansions of the force function of the Newtonian interaction of the body with the surrounding celestial bodies in "action-angle" variables. The first attempts to construct such expansions in the theory of rotation of the Earth have been made in the work of H. Kinoshita (1977) for the second harmonic of the force function. In this case the author has made some simplifications and some restrictions on the dynamic oblatenesses of the planet. But it was in the works of Yu Barkin (1992, 1998), these expansions were obtained in an exact representation in the form of Fourier series on multiple variables "angle" with the coefficients represented by functions of the variables "action." This representation of the coefficients is not trivial, and makes extensive use of the apparatus of elliptic functions, theta functions, hyperbolic functions and elliptic integrals of three kinds.

In the papers (Yu. Barkin, 1992, 1998), the expansion of the force function was obtained for the main terms of the second harmonic proportional to the coefficients of the gravitational field of the body C₂₀ (zonal harmonic) and C₂₂ (the main tesseral harmonic). In this paper, a complete expansions in Fourier series of all components of the second harmonic of the gravitational potential, i.e. additional terms proportional to the geopotential coefficients C₂₁, S₂₁ and S₂₂ have been obtained (M. Barkin, 2011). The expansions are presented in a compact form convenient for applications.

Analytical formulas for the first-order perturbations in "action-angle" variables in this more general formulation of the perturbed rotational motion of the planet (the Earth) under the gravitational attraction of external celestial bodies (Moon, Sun) have been obtained in explicit form. We studied also the changeable Earth's rotation with variable coefficients of the gravitational potential C₂₀ (t), C₂₂ (t), C₂₁ (t), S₂₁ (t) and S₂₂ (t). Separately the dynamic effects due to the observed secular changes of these coefficients and the observed annual variations obtained by the long-term observations of geodetic satellites (Cheng et al., 1997, 1999) have been studied.

Follow to modern studies and works of astrophysicists (Link et al., 2001) we developed a number of applications of the formulas of our unperturbed rotational motion to study the kinematics and dynamics of isolated pulsars with precession.

Keywords: Kinoshita variables, Sadov variables, unperturbed and perturbed motion, gravitational attraction, celestial bodies, <<action-angle>> variables