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Green's function in the non-integer dimensional space: the fractional Laplacian for the fault zone

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Generally, the crust materials have a various scale of the discontinuous property. Since the effect of the discontinuity can be described quantitatively by the non-interger property of the space (e.g., Fractal geometry), we consider the Green's function in the non-interger space. Especially, we take up the fractional Laplacian as follows:

(-¥Laplacian)$^{a/2}$ $f(r)$=-$g(r)$ (1)

where a is the order of the differentiation and the real number. When a=2, this is the ordinary Laplacian, therefore, f is, for instance, the volumetric strain or the trace of the stress tensor and so on; g is the perturbative force. Except when a is an even number, the distance dependence of the Green's function is given by the Riesz potential:

G ~ r^{a-2} (2)

From this relationship and the Laplacian of the dimension D, we obtain

D+a=4 (3)

except when a is an even number. For instance, when a=1 and 3, we have G ~ 1/r and G ~ r, respectively. These are the well-known results in the thee-dimensional and the one-dimensional space. Note that since a is the real number, Eq. (3) shows that the dimension D can also take a non-interger values. For instance, when a=1.5, i.e., D=2.5 dimensional space, the Green's function take the form: G ~ $r^{-0.5}$.Since a itself is defined as the order of the differentiation, the larger number of a means that the problem, described by the fractional Laplacian, becomes more non-local. Eq. (3) shows that the order of the differential is inversely proportional to the number of the dimension of the space. In fact, as the D grows larger, the problem becomes more local. This is a phenological interpretation of Eq. (3).Our conclusions are as follows: (1) In the discontinuous space such as a fault zone, the order of the fractional differentiation has a relationship with the number of the dimension of the space. (2) This relationship determines the concrete form of the distance dependency for the Green's function in the non-integer dimensional space.

Keywords: non-integer dimension, Green's function, Fractional Laplacian, Fault zone