

## Phenomenological constitutive law for transient rheological behavior of rocks and minerals

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Phenomenological constitutive law for transient rheological behavior of rocks and minerals involving the work hardening effect is investigated in terms of viscoelastic theory and fractional rheology. Viscoelastic behavior is often defined in term of the linear response function, and output of stress is the convolution integral of viscoelastic response and strain-input. The response function is called relaxation modulus (a time-dependent modulus of stress to strain) and regarded as a constitutive equation. The convolution integral is identical to a definition function of fractional-order derivative, and stress is then equal to the noninteger-order derivative of strain, which expresses the behavior between Hookean elasticity and Newtonian viscosity. Using the constitutive law, we analyze the experimental data of high-temperature deformation of rocks and minerals such as marble, halite and orthopyroxene. The relaxation modulus of rocks and minerals shows a temporal power-law scaling, and the exponents of the power-law corresponds to the reciprocal of exponent of stress in the flow law. The constitutive equation then represents both transient and steady-state behaviors in the same mathematical structure. The exponents span the range from 0.04 to 0.13 (the stress exponent, from 7.5 to 25.0) for the transient behaviors and from 0.14 to 0.25 (the stress exponent, from 4.1 to 7.1) for the steady-state behaviors. The exponent of stress in the flow law is dependent on the deformation mechanisms such as diffusion and dislocation creeps. For orthopyroxene, the response function involves the strain nonlinear function, a power-law of strain, expressing the effect of work hardening, and can be transformed into the empirical evolution equation of work hardening, stress equals to the power-law of strain and strain-rate.

Keywords: viscoelasticity, fractional calculus, transient behavior, rheology, work hardening