

Kaula's rule and the scaling law of the Kaula constant in the lunar-planetary gravity fields

Mina Hashimoto^{1*}, Kosuke Heki¹

¹Department of Natural History Sciences, Faculty of Science, Space Geodesy, Hokkaido University

Gravity fields of the moon and planets are modeled as the sum of spherical harmonics of various degrees/orders, and their coefficients are called as the Stokes' coefficients. These coefficients with degrees and orders complete to a few hundreds have been estimated using the tracking data of artificial satellites such as GRACE (the earth), SELENE and GRAIL (the moon). High degree coefficients show fine structure of the shallow mass distribution, and low degree coefficients reflect global scale mass distribution of the body. Kaula's rule-of-thumb predicts that the Stokes' coefficients are inversely proportional to the square of the degree n of the spherical harmonics. In this study, I confirmed that this is the case for the moon, the earth, Mars and Venus. Smaller coefficients for higher degrees mean that the long wavelength components have larger amplitudes.

Here I refer to the factor to link $1/n^2$ to the Stokes' coefficients as the Kaula constant. The smaller celestial body is considered to have a larger Kaula constant, and they are considered to obey a scaling law that the coefficient is inversely proportional to the square of the surface gravity of the body (in the original paper by Kaula [Kauka,1963], the constant is suggested to scale with R^4/M^2 , where R and M denote the radius and the mass of the body, respectively). This scaling law is confirmed to hold true for the moon, Mars, Venus, and the earth. Departure from this scaling law would imply some difference of the physical properties (such as viscosity) of the material that makes up the interior of the body. Recent data on the gravity field of Mercury taken by MESSENGER seem to indicate such a departure, which may reflect the unusually large relative radius of the metallic core of Mercury.

The lunar farside and nearside are known to be very different, i.e. the nearside has thin crust and flat terrain, whereas the farside has thick crust and rugged terrain. There are several hypotheses for the origin of such lunar dichotomy, and many of those suggest some difference in thermal history between the two sides. Such a difference can be studied with the gravitational field. Here I compared the Kaula constant of these lunar two hemispheres by creating two hypothetical moons, those composed of only farside and only nearside. The Kaula constant of the farside showed slightly larger value than the nearside, suggesting colder internal temperature of the lunar farside.

Keywords: scaling law of the kaula constant