Japan Geoscience Union Meeting 2013
(May 19-24 2013 at Makuhari, Chiba, Japan)

SVC50-01 Room:106 Time:May 20 11:00-11:15

An analytical formula for the longitudinal resonance frequencies of a fluid-filled crack

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The fluid-filled crack model (Chouet, 1986) has been most commonly used to interpret oscillation frequencies of volcanic earthquakes. Kumagai and Chouet (2000) systematically studied the complex frequencies of resonances of a crack filled with various kinds of fluids. Following their study, the complex frequencies of LP and VLP events at volcanoes have been linked to fluids and geometries of the cracks. So far, the crack model simulations have been performed using the finite-difference (Chouet, 1986) and boundary integral (Yamamoto and Kawakatsu, 2008) methods. These methods require computationally extensive procedures to estimate the complex frequencies of crack resonance modes. Establishing an easier way to calculate the frequencies of crack resonances would help to interpret the observed frequencies. In this study, we demonstrate that the longitudinal resonance frequencies of a fluid-filled crack can be described by an analytical formula.

We consider a 1D longitudinal oscillation of a fluid-filled crack. The fluid pressure averaged through the crack aperture, denoted as P, satisfies the following relation (Kumagai, 2009)

$(d^2/dt^2)[P(x,t)+(2b/d)u_d(x,t)]=a^2(d^2/dx^2)P(x,t)$, (1)

where a and b are the sound speed and bulk modulus of the fluid, d is the crack aperture, and u_d is the displacement on the crack surface. To derive the crack wave velocity from Eq. (1), a relation between P and u_d is required. Kumagai (2009) assumed a proportional relation between P and u_d to derive the velocity. We computed P and u_d using the FDM code of Chouet (1986), which indicated that P is proportional to u_d in time but not in space; rather the ratio u_d/P showed an ellipsoidal spatial distribution. Inserting this relation into Eq. (1) yielded a 1D variable-coefficient partial differential equation, which we semi-analytically solved to obtain a formula

$f_m=(m\text{-}1)a/[2L\{1+2e_m(b/G)(L/d)\}^{1/2}]$, (2)

where f_m is the frequency of an oscillation mode of a wavelength $2L/m$, m is an integer, G is the rigidity of the solid, L is the crack length, and e_m is a constant which depends on the oscillation mode. To check Eq. (2), we computed the oscillation frequencies for various L/d using the FDM code of Chouet (1986). The results were in good agreement with Eq. (2), suggesting that the equation adequately describes the frequency.

Eq. (2) relates the frequency f_m to the fluid properties a and b as well as the crack geometry parameters L and d. We used the equation to interpret a swarm of more than 40,000 LP events with an almost constant frequency of 0.7-0.9 Hz observed at Taal volcano, Philippines. Our waveform analyses of the events suggested a vapor-filled crack for which the fluid properties were kept constant. However, an inflow vapor volume variation causes a crack geometry change. Thus a frequency variation due to the geometry change may occur, whereas the observed frequencies were almost constant. Eq. (2) indicates that f_m is proportional to $(d/L^3)^{1/2}$ for large L/d, which suggests a constant oscillation frequency if d is proportional to L^3. We consider the crack geometry controlled by a balance between buoyancy and elastic forces. In this case, d is proportional to L^2. Assuming this relation and a vapor temperature of 600 K under a pressure of 5 MPa for the LP events at Taal, we estimated that the observed frequency of 0.7-0.9 Hz can be explained by a crack volume variation by a factor of 4. This suggests that a certain range of the inflow vapor volume variation is possible for the almost constant frequency.

The analytical formula obtained by this study may have a wide applicability to interpret oscillation frequencies of LP and VLP events at other volcanoes.

References
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Keywords: Fluid-filled crack model, LP events, Resonant frequency, Taal volcano