

THE THEORY OF THE MOON ROTATION AND DETERMINATION OF FOURTH MODE ITS FREE PHYSICAL LIBRATION

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Resume. In result of the comparison of our analytical theory of lunar physical libration, in particular the free librations, and available now, the empirical theory of the Moon's rotation (Rambaux, Williams, 2011), we have identified period, amplitude, and the initial phase of the forth mode of free libration of the Moon, caused by liquid core.

Analytical theory. The main study here is a construction and development of a highly accurate analytical theory of physical libration of the two-layer Moon (with uniform ellipsoidal liquid core and non-spherical elastic mantle). The core of the Moon is modeled by ellipsoid with an ideal homogeneous fluid. The mantle is considered as non-spherical solid body. The theory is developed on the basis of the canonical equations in Andoyer - Poincare variables and by special methods of the perturbation theory on construction of quasi-periodic solutions and investigation of their vicinity (based on the relevant equations in variations). The tables of values of the amplitudes and periods of forced and free librations for Andoyer ? Poincare variables describing the libration of the Moon and the core, for the variations of the components of the angular velocity of rotation of the Moon and the angular velocity of rotation of the coordinate system of Poincare (with respect to which a simple fluid motion is determined) have been obtained and studied. In first we have studied contributions in librations of the Moon of the second harmonic of selenopotential in accordance with the modern Selena model of gravitational field of the Moon (Matsumoto et al., 2010). The novelty of the theory and its practical significance are determined by the following principal provisions:

1.New forms of equations of physical libration of the two-layer model of the Moon (in particular in Andoyer ? Poincare variables) and new methods for their study; 2.Highly accurate description of the developments of spherical functions of the coordinates of the Moon, in the expression of the force function; 3.The new two-layer Mizusawa model of the Moon and Selena model of the gravitational field of the Moon; 4.Cassini rotation of the Moon, forced and free librations of the Moon in analytical form and their tables; 5.Dynamical effects in forced and in free librations caused by a liquid core; 6.Dynamical effects in forced and free librations of the Moon caused by its elasticity; 7.Determination of the forth mode of free libration caused by the liquid core; 8.Identification of some terms of modern Rambaux-Williams empirical theory.

Determination of the period, amplitude and phase of the fourth mode of the free libration of the Moon caused by the liquid core. We have been compare free libration terms from our analytical theory with some unidentified terms from empirical theory (Rambaux, Williams, 2011). In results 8 unidentified terms for classical variables in empirical theory were explained and amplitude, initial phase of the Moon free libration have been determined. The period of free libration of the pole of the Moon with liquid ellipsoidal core appreciated by us in 205.7 yr. The amplitude and initial phase of Poincare long-periodic argument of the free libration in pole motion due to liquid core have been determined in 0°0395 and -134 degrees (for initial epoch 2000.0 JD). In accordance with developed analytical theory this period corresponds to the sum of dynamic compressions of the core in 7.24×10^{-4} , that is in agree with seismographic data and data of laser observations (Barkin, Hanada et al., 2012). In assumption about similarity of ellipsoidal core and the entire Moon we have obtained the estimations of oblatenesses of the liquid core: 4.42×10^{-4} and 2.83×10^{-4} .

References:

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