An arithmetic geometrical approach for modeling of seismic activity

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Earthquake occurs in a discrete manner in time and space. Except for the main shock-aftershock sequences, earthquakes that seem to be individually independent. However, when viewed as a whole, we find that there are laws, for example, G-R law, that govern the entire earthquakes that seem to be individually independent.

A similar phenomenon can be observed also in the world of "number". The most basic example is the distribution of the prime numbers in integers. The presence of interesting relationships, for example, the reciprocity law of quadratic residue, has been found in prime numbers that seem to be individually independent. Class field theory as a theoretical system relates to the structure of the number has been established. In addition, a lot of knowledge is going to be achieved in recent years by the development of arithmetic geometry that regards "number" as geometric objects. Arithmetic geometry was developed by applying the theory of scheme to "number", which was developed in algebraic geometry aimed to study algebraic varieties consisting of zeros of polynomials. Arithmetic geometry have revealed interesting laws that govern the world of the "number".

We consider the correspondence as follows to the world of "number" and "earthquake" that seem to be completely independent. Let p_i be the i-th prime and take appearance interval of prime, p_i-p_{i-1}, as index of i-th prime. Let e_i be the i-th earthquake, $T(e_i)$ be occurrence time of the i-th earthquake, and $M_o(e_i)$ be moment of the i-th earthquake. We assume that the following equations hold.

$T(e_i)=p_i$,

$\log(M_o(e_i))=p_i$-p_{i-1}

By performing numerical experiments, we have found that the characteristics of this correspondence are similar to the G-R law. We call the model obtained by this correspondence as "arithmetical seismic activity model".

Based on these considerations, we can regard the "arithmetical seismic activity model" as an object of the scheme theory. By using knowledge from the arithmetic geometry, it is expected to obtain knowledge about laws that control seismic activity. We have an image of earthquakes that fracture starts from a point in space that does not have infinite special boundary conditions and expand gradually to the destruction of the entire area. Although it is believed that the boundary conditions more realistic for the physical model, we expect to be able to handle them by considering to limit or to extend the "number" and those are objects of the scheme theory. By using the "arithmetical seismic activity model", the unresolved issue on the math as "the twin primes exist infinitely" is replaced by the problem of the nature of the seismic activity, such that the smallest earthquakes occur infinitely.

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