

Volume source representations: a possible unified explanation based on the representation theorem

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The moment tensor inversion is a powerful tool to extract source information from seismic and geodetic observations. A moment tensor for earthquake faulting has been determined and its non-diagonal components give the seismic moment (rigidity \times slip \times fault area), which is one of essential source parameters of an earthquake. The sum of the diagonal components (the trace) of a moment tensor represents volumetric change at the source. A moment tensor determined for a volcanic source frequently has non-zero trace. However, it has been failed to uniquely relate the diagonal components to the actual volume change, which remains a critical issue in volcanic seismology (Kazahaya et al., 2011). For example, two different volume changes DV and dV have been proposed for the seismic moment of a spherical source geometry; DV comes from the moment tensor definition of a seismic fault having opening displacements whereas dV is obtained from the equivalence of resultant displacement fields due to the former moment tensor and an isotropically expanding sphere in an elastostatic equilibrium.

The difference between DV and dV has been discussed in the last decade. Muller (2001) considered an open crack of a spherical shell shape and showed that DV is the volume of the opening and dV is only the part opening outward. Aki and Richards (2002) and Richards and Kim (2005) adopted Eshelby's approach which considers virtual operations consisting of cutting, stress-free transformation, elastic straining, and welding, and concluded that the difference is due to whether the volumetric change occurs in an unconfined condition (DV) or in a confined condition (dV). Kumagai et al. (2013) reconsidered this problem and concluded that the displacement field due to a spherical source does not coincide with that due to a three-perpendicular-crack source though they both are represented by isotropic moment tensors. They also extended the insights into sources in a bimaterial medium. It is worth mentioning that the approaches of AR2002 and RK2005 give a conceptual explanation on how to adjust DV to the actual volume change dV for a sphere, but not for arbitrarily shapes. Here we address how to make such adjustments for general geometries on the basis of the representation theorem. Our imaginary operation below gives a unified explanation for the two different volumetric changes and newly proposes a method of estimating dV of the inversion results for arbitrary source geometries.

We start with the representation theorem that gives the displacement field by two terms (without a body force): a surface integration on the source region with convolution of the surface displacement and the gradient of the Green's function normal to the surface (1), and that with convolution of the surface traction and the Green's function (2). Only (1) has been considered for the seismic fault because (2) vanishes due to the balance of the traction at the contacting surfaces of a fault. On the other hand, (2) does not vanish in the case of a volumetric source, and therefore a quantitative adjustment is required to include the effect of (2) into (1). We here demonstrate that such an adjustment, is always realized by introducing an additional imaginary volumetric change, which works as 'displacement glut' in our representation of moment tensor. Our representation is found to be mathematically equivalent to the rather conceptual 'stress glut' representation proposed by Backus and Mulcahy (1976). We present a unified explanation for the existing various representations and propose a method to practically evaluate the moment tensor components from the boundary conditions of the volumetric source. The proposed representation will be useful in connecting dynamical models of volcanic processes and moment tensors.

Keywords: Moment tensor, Volumetric source, Representation theorem, Green's function, Volcano seismology, Explosion source