A divergence-form wave-induced pressure for extending the Eliassen-Palm theory to all waves at all latitudes

AIKI, Hidenori; TAKAYA, Koutarou; GREATBATCH, Richard

1Japan Agency for Marine-Earth Science and Technology, 2Kyoto Sangyou University, 3GEOMAR / University of Kiel

Classical theory concerning the Eliassen-Palm relation is extended in this study to allow for a unified treatment of mid-latitude inertia-gravity waves (MIGWs), mid-latitude Rossby waves (MRWs), and equatorial waves (EQWs). A conservation equation for (what the authors call) the impulse-bolus (IB) pseudomomentum is useful because it is applicable to ageostrophic waves and the associated three-dimensional flux is parallel to the direction of the group velocity of MRWs. The equation has previously been derived in an isentropic coordinate system or a shallow water model. The authors make an explicit comparison of prognostic equations for the IB pseudomomentum vector and the classical energy-based (CE) pseudomomentum vector, assuming inviscid linear waves in a sufficiently-weak mean flow, to provide a basis for the former quantity to be used in an Eulerian time-mean (EM) framework. The authors investigate what makes the three-dimensional fluxes in the IB and CE pseudomomentum equations look in different directions. It is found that the two fluxes are linked by a gauge transformation, previously unmentioned, associated with a divergence-form wave-induced pressure (symbolized as $\Lambda$ in the present study). The quantity $\Lambda$ vanishes for MIGWs and becomes nonzero for MRWs and EQWs, and may be estimated using the virial theorem. Concerning the effect of waves on the mean flow, the quantity $\Lambda$ represents an additional effect in the pressure gradient term of both (the three-dimensional versions of) the transformed EM momentum equations and the merged form of the EM momentum equations (the latter of which is associated with the nonacceleration theorem).

http://dx.doi.org/10.1175/JAS-D-14-0172.1

Keywords: inertia-gravity waves, mid-latitude Rossby waves, equatorial waves