

Variable inertia method: A novel numerical method for mantle convection simulation

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It is important to understand mantle convection in the earth and the 3D numerical simulation has been a very useful tool. In almost all previous simulations of the mantle convection, the anelastic approximation and the (extended) Boussinesq approximation has been used. An implicit method should be used to solve the equation of motion applied above approximations to allow the use of long timesteps not limited by the CFL condition. However, the resulting matrix is ill-conditioned, in particular since the viscosity depends strongly on the temperature. Thus, 3D simulation of the mantle convection is not well-suited to modern large-scale parallel machines. We have developed an explicit method which can be used to solve mantle convection problem.

Recently, Rempel (2005) proposed the reduced speed of sound technique (RSST) to solve the thermal convection of our Sun. Previously, an incompressible approximation was used to solve the convection of the Sun, as a result, an implicit method was used. The basic idea of RSST is, as its name suggest, modify the continuity equation to reduce the sound speed, thereby allowing the long timestep not limited by the physical CFL condition. Hotta et al. (2012) applied RSST to the thermal convection of our Sun. They confirmed that the characteristic of the flow is not changed as far as the Mach number is smaller than ~ 0.7 , and successfully performed the high resolution simulation of our Sun (Hotta et al. 2014, 2015).

Since not only Mach number, but also the Reynolds number is very small in the mantle, we developed a new formulation which can change these two numbers independently. In order to reduce the sound speed, we multiplied the inertia term of the equation of motion by a large and viscosity-dependent coefficient. We also scale the thermal conductivity and the viscosity in a consistent way so that the characteristic of the flow is unchanged. Using these two modifications, we can change the Mach number and Reynolds number independently. Theoretically, we can expect that this modification would not change the flow as far as the Reynolds number and the Mach number are sufficiently smaller than unity. We call this method the variable inertia method (VIM).

We have performed an extensive set of numerical tests of the proposed method for thermal convection, and confirmed that the characteristics of the flow are not changed by VIM. In particular, it can handle the difference of viscosity of more than five orders of magnitude. Since we can use the long timestep with VIM, we can finish a simulation much faster than that without VIM without losing accuracy. Our new scheme is well suited for simulations on modern large-scale parallel machines since it is an explicit method and thus it opens a new door to very high resolution simulations of mantle convection.

Keywords: mantle convection, numerical simulation