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Waveform inversion converged towards the grand minimum: A Wiener-filter approach

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Not only in seismology but also in many research fields of earth and planetary sciences, waveform inversions are widely used. Some on-going data analyses even adopt automated inversion procedures with huge amounts of data. Their basic concept is the minimization of the difference between each observed time-domain waveform and its synthetic waveform in the least squares sense. Since waveforms are characterized by a series of peaks and troughs, there are many local minimums around the true or grand minimum so that we may obtain our final inverted model for one of local minimums rather than the desired grand one particularly if our initial model for inversion is not close to the true model. In a mathematical point of view, this problem is related to the ambiguity of 2π in phase. The general process to overcome the above problem is called 'phase unwrapping', but this process is rather empirical, very subjective and different from a problem to another.

We propose a new inversion approach in order to make waveform inversions objective and stable. This consists of two stages. At first, the difference between each observed waveform $d(t)$ and synthetic one $p(t)$ (here we shall assume that all the waveforms are expressed by discrete time series) by a Wiener filter $w(t)$. The Wiener filter is defined by the filter that minimize the sum of squares of $p(t){*}w(t)$-$d(t)$, where $*$ represents the convolution. Coefficient of w(t) can be obtained efficiently by a recursive algorithm (Levinson, 1949). If $p(t)$ and $d(t)$ were same, the Wiener filter $w(t)$ would be a unit filter, that is, 1 for t=0 and zero otherwise (corresponding to a delta function in a continuous case). In the second step, therefore, we introduce a new criterion to minimize the sum of squares of $w(t)$ multiplied by a time lag for all the records together: $t \bullet w(t)$. In practice, we need to normalize the above criterion divided by the square of $w(t)$.

With this new criterion, the time lag of $w(t)$ is only weighted without its sign (i.e., positive peak versus negative trough), there are no ambiguities in phase, unlike any conventional waveform inversions. If we iterate the above inversion procedure by modifying a given model in each step, our model will be converged towards a optimal one without falling into any local minimums.

For large-scale inversion problems, one uses an adjoint matrix or operator rather than the formal inverse matrix of a given forward problem. The modification of a model in each iteration step is obtained by the data residual, $p(t)$ – $d(t)$, multiplied by the adjoint matrix. The adjoint matrix is derived by the partial derivative of by the minimizing criterion (the sum of squares of $t \bullet w(t)$ in this case) partially differentiated by the synthetic $p(t)$. Its result is expressed by - $t^2 \bullet w(t)^2 * p(t)^{-1}$. That is , we only need to deconvolve the sum of squares of $t \bullet w(t)$ by $p(t)$. Other procedures are similar to those of conventional waveform inversions.

We conducted numerical tests in order to confirm the efficiency of our new inversion approach, using a medium with velocity fluctuations. Even without a dense coverage of paths connecting sources and receivers, our inversion leads to a final model close to the true model. In contrast, an initial model very close to the true one is required if we obtain a satisfactory model with conventional least-squares criteria in inversion. That is, the present inversion approach can lead our model search into the true or grand minimum without falling into or being stuck by local minimums around it.

Keywords: inverse problem, seismic waveform, Wiener filter, grand minimum, phase unwrapping