Envelopes in 3-D Random Media: Comparison of the new Markov approximation and the finite difference simulation

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Short-period seismograms show complex waveforms reflecting small scale heterogeneities in the earth. For example, the main part around the peak of the seismograms becomes broader than the source duration and long lasting coda waves are excited. We can interpret these phenomena by considering the wave propagation through the random velocity fluctuated medium based on the scattering theory. There are several stochastic methods such as the radiative transfer theory, diffusion approximation and the Markov approximation. In this study, we focus on the Markov approximation based on the parabolic approximation. The applicable range of the Markov approximation has been validated by comparing with the finite difference (FD) simulation of the wave equation in 2D random media. However, the validation in the 3D random media is limited. Taking the wavenumber of a wavelet as a reference, Sato (2016) proposed an extension of the Markov approximation by dividing the spectrum of the fluctuation into the long-scale which contributes the envelope broadening and small-scale component which affect the attenuation. By applying the Markov approximation to the long-scale component, we can obtain the analytical solution of the mean square (MS) envelope for random media having the power-law type spectrum of the fluctuation. Here, conducting 3D FD simulations, we seek the applicable range of the new Markov approximation by comparing the theoretical envelopes with FD simulation envelopes.

In the FD simulation we use 1.5 Hz and 3.0 Hz Ricker wavelets. We assume that the average velocity is 4 km/s. The grid spacing is 40 m and the time interval is 3 ms. We solve the 3D scalar wave equation with the 4th and 2nd order accuracies for the space and the time, respectively. The medium size for x and y directions are 174 km and that for z direction is 250 km. For the 1.5 Hz case, we double the grid spacing and time interval and half the number of the grid for each direction. We conduct at least 5 FD simulations with different random seeds and set 9 receivers at each propagation distance. Therefore we stack at least 45 envelopes to calculate the MS envelope.

We assume that the correlation distance is 5 km and the root mean square of the fluctuation is 5 %. We change the κwhich controls the roll-off of the power spectrum for 0.1, 0.5 and 1.0. For 3 Hz simulation, we set κ=0.5.

The duration of the MS envelopes derived by the FD simulation becomes broad with the propagation distance. The peak amplitude of the MS envelope decreases as $r^{-2.6}$ to $r^{-3.5}$. This decay rate is large for a small κ. For the case of the 3 Hz, the decay is $r^{-2.7}$. The excitation of the coda is strong for a small κ.

Sato (2016) define the corner wavenumber of the small-scale component as $1/a_s = \zeta k_c$, where k_c is the central wavenumber of the wavelet. Hence $\zeta$is a fine tuning parameter of the reference scale. For the case of the small ζ, the contribution of the small scale component becomes large. The envelope broadening is weak and the scattering attenuation is strong. The envelope duration becomes long and the scattering attenuation becomes small for large ζ. We found that the theoretical envelopes are well fitted to the FD envelopes for ζ=1.0 and 1.5. Even when κ=0.1, the theoretical envelopes can roughly model the FD envelopes. However, the decay rate with the propagation distance of the FD envelopes is larger than that of the theoretical envelopes for all values of ζ.

We will examine the validity of the new theory for different $\kappa$and center frequencies in the future. We will analyze the fluctuation of MS envelopes of the FD simulation, too.

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