

The difference between JMA magnitude and moment magnitude in terms of seismic efficiency

*Kiyohiko Yamamoto

1. Introduction: For large earthquakes occurring along the Japan trench except for the off Miyagi Pref. region, moment magnitude M_w is 0.4 larger than the JMA magnitude M_j . Here, M_j and M_w are proportional to logarithms of seismic energy E_s and released moment M_o , respectively. M_o is proportional to the relative displacement u_b of fault surfaces. E_s depends on directly seismic efficiency f , but M_o does indirectly. Thus the difference is thought to reflect the degree of the dependence on f .

f is a function of the rupture velocity V_r and is large for a large V_r . The small M_j compared with M_w thus suggests the small V_r . The difference between M_j and M_w is discussed from this viewpoint for The Tohoku earthquake (2011/3/11, M_w9) as an example.

2.Theory: The damage zone fault model of earthquakes* is employed for the present discussion. In this model, a fault zone with a uniform thickness constitutes of damaged rock area and asperity area. Fault surfaces mean the boundaries between fault zone and host rock blocks. The damaged rocks have relaxed during a long time after the preceding faulting. An asperity has the same elastic constants as the host rocks. Faulting occurs at the time that the relative displacement u_b reaches the critical value.

For faulting, energy balance is written by

$$P_a + P_b = E_s + W, \quad E_s = f \times P_b. \quad (1)$$

Here, P_a and P_b respectively are strain energies in the asperity and in the host rock blocks. P_b is approximated by the strain energy released when a circular crack is produced in a homogeneous host rocks under the uniform stress, that is equal in magnitude to the average stress drop due to faulting.

W is apparent fracture energy that is equivalent to the work to the host rocks done by the vertical displacement of the fault surfaces. The displacement is produced by the rotation of damaged rocks accompanied by the rupture propagation in a fault zone.

A linear relation has been found between the width of fault damage zone and the length of fault (Vermilye, J. M., and C. H. Scholz, 1998). In order to link the model to fault size, the linear relation is adopted.* Further, Sato and Hirasawa (1973) present an approximate relationship between V_r and f for a circular crack. This relationship is used for the present discussion.

3. Results: For $f=1$, all strain energy P_b is dissipated as E_s . The fraction of asperity area is about 2% of the fault zone area. V_r is approximately equal to the S-wave velocity of host rocks. For f close to zero, all strain energy $P_a + P_b$ is used for the rupture propagation and E_s and V_r go to zeros. These may be the characteristics of so-called slow slip events. The fraction tends to about 0.74%. This is 0.37 times of the fraction at $f=1$. This means that the displacement and the average stress drop decrease to 0.37 times of those at $f=1$.

The relationships between E_s and M_j and between M_o and M_w respectively are written by

$$\log E_s = 1.5M_j + 4.8 \quad (2)$$

$$\log M_o = 1.5M_w + 9.1. \quad (3)$$

For a constant fault area, Eq. (2) and Eq. (3) intersect around $f = 0.8$. This suggests that the seismic efficiency is about 0.8 for majority of earthquakes. For $f = 0.8$, V_r is determined at about 0.8 times of S-wave velocity V_s of host rock.

Referring to the report by JMA**, M_j and M_w of The Tohoku earthquake are 8.4 and 9.0, respectively, and V_r and V_s are about 1.8km/s and about 3.4 km/s, respectively. V_r is about 0.53 times of V_s . From the relationship of V_r and f , f is estimated at about 0.3. M_j is estimated at about 8.6 for f

= 0.3 and $M_w=9.0$. The estimated M_j tends to the observed one. This suggests that the small M_j is due to the small V_r .

Note: *Yamamoto and Yabe, 2009; <http://kynmt.in.coocan.jp/> ;(REFERENCE/23)

**<http://www.jma.go.jp/jma/kishou/books/gizyutu/133/ALL.pdf>

Keywords: Seismic efficiency, Moment magnitude, JMA magnitude, Slow slip event, Rupture velocity, Damagezone fault model of earthquake