Mathematical formulation of forecasting volcanic eruption sequence based on physical models and field observations

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During a volcanic eruption, the intensity and style of eruption generally change with time. In order to establish a method to forecast such eruption sequences, forward and inverse problems are mathematically formulated on the basis of physical models for the magma plumbing system including the conduit flow dynamics and the magma chamber processes.
The variation of eruption sequence is characterized by how magma discharge rate, Q, changes with time as a function of magma chamber pressure, P. According to the conduit flow models, the qualitative feature of the relationship between the magma discharge rate and the chamber pressure (the Q-P relationship) during explosive eruptions is controlled by the pressure at which the conduit flow changes from a bubbly flow to a gas-pyroclast flow (i.e., the fragmentation pressure). The fragmentation pressure, in turn, depends on the mechanisms of gas-escape and magma fragmentation. The Q-P relationship during non-explosive (effusive) eruptions depends on the density change due to gas-escape process and the viscosity change due to crystallization during magma ascent in the conduit. The physical model of magma chamber processes, on the other hand, suggests that the Q-P relationship is affected by the effective compressibility and volume of magma chamber. The effective compressibility of magma chamber drastically increases when the magma contains gas phase, and hence, it depends on water content and pressure of magma. Because of the coupled effects of conduit flow dynamics and magma chamber processes, the forward model of the magma plumbing system shows diverse behavior of eruption sequences (i.e., various trajectories of the Q-P relationship).
In order to forecast the eruption sequence, we must estimate the values of parameters that control the trajectories of the Q-P relationship in the above forward model. Generally, the inverse problem of the magma plumbing system is formulated as a problem to estimate the product of volume and pressure change of magma chamber and the effective compressibility of magma chamber from the field data on ground deformation around the volcano and mass of the erupted magma. The estimation of the rest of the parameters (e.g., the density and the viscosity of magma in the conduit) requires additional field observations such as petrological data of erupted magma. The numbers and kinds of parameters that can be estimated from the inverse model depend on the mathematical characteristics of the conduit flow model. For effusive eruptions where the conduit flow is approximated by a Poiseuille flow so that P and Q are proportional, a parameter expressed by the combination of viscosity, conduit length, conduit radius, chamber volume and effective compressibility of magma chamber is collectively estimated from the decay constant of Q and P during the waning stage of the eruption. For a certain type of explosive eruptions, on the other hand, the value of the fragmentation pressure can be constrained by the trajectory of the Q-P relationship observed during the waning stage of the eruption. In the presentation, we mainly discuss how the uncertainties of the parameter estimation and the forecast of eruption sequence depend on the mathematical characteristics of the conduit flow model.

Keywords: volcanic eruption sequence, physical model, magma plumbing system

